

# Bias in CDMA Channel Estimates With the Use of Short Spreading Sequences

Teng Joon Lim<sup>1</sup>

Centre for Wireless Communications  
National University of Singapore  
Singapore 117674.  
E-mail: limtj@cvc.nus.edu.sg

**Abstract** — We demonstrate that channel estimates obtained using the conventional correlation-based approach are biased when short spreading sequences are employed in CDMA systems. A simple decorrelating estimator is proposed to remove the bias and hence improve estimation performance.

## I. INTRODUCTION

Channel estimation is required for coherent detection in most code division multiple access (CDMA) receivers, including the Rake detector [1], the blind adaptive detector of [2], and multi-stage interference cancellers (e.g. [3]). Inaccurate channel estimation will adversely affect their performance, especially the multiuser detectors' [4, 5].

In practice, the channel response is usually estimated with the aid of pilot symbols, through correlation with a pilot spreading code synchronized to the multipath delays, which are assumed to be estimated using a separate delay-tracking device. More complicated methods, based for instance on subspace estimation (e.g. [6]), have also been proposed but will not be considered.

In this paper, we show that correlation-based channel estimates are biased when short spreading codes (i.e. those with a period of one symbol interval) are used. Short codes are becoming important<sup>1</sup> because they make certain multiuser detection techniques less complex (e.g. the decorrelator) while others (e.g. those based on identifying signal and noise subspaces [8]) need the time-invariant or cyclo-stationarity property of short codes to function. While detectors implemented using adaptive filters [7] do not require channel estimates, most others do and therefore channel estimation bias is a potentially serious problem that should be studied more thoroughly.

To remove the bias, we introduce a simple decorrelating estimator that requires an additional  $L^2$  multiplications compared to the conventional estimator, where  $L$  is the number of paths tracked by the receiver. Simulation results show that the proposed estimator reduces channel estimation error and improves bit error rate (BER) substantially.

## II. TIME MULTIPLEXED PILOTS

We consider the synchronous downlink channel in this section, but it should be apparent shortly that our results hold for the

<sup>1</sup> This work was supported by the National Science and Technology Board, Singapore.

<sup>2</sup> For instance it is an optional feature on the uplink of the IMT-DS air interface for third-generation cellular systems.

asynchronous uplink as well. The DS-SS signal (sampled at chip rate) transmitted on the  $k$ th code channel is given by

$$x_k(n) = \sum_{i=1}^{M-1} d_k(i) s_k(n - iN) \quad (1)$$

where  $d_k(i)$  is the  $k$ th symbol stream,  $s_k(n)$  is the short spreading code used in the  $k$ th channel and is non-zero only in the range  $0 \leq n < N$ ,  $N$  is the processing gain and  $M$  is the total number of symbols transmitted in a packet. There are  $K$  code channels in total, and without loss of generality, the first one  $s_1(n)$  carries the pilot symbols, meaning that  $P$  of the  $M$  symbols  $d_1(i)$  are known to the receiver. The various code channels may be assigned to different users, or to only one high-rate user, without affecting our conclusions.

The channel is assumed to have  $L$  paths, characterised by time delays  $\tau_l$  (which are in terms of sampling intervals) and complex attenuations  $c_l(i)$ ,  $l = 1, \dots, L$ . This representation does not restrict the applicability of correlation-based channel estimation to slowly fading channels. However it does highlight the fact that this approach to channel estimation can deliver only one estimate per symbol interval – if the maximum Doppler frequency is comparable to the symbol rate, then we will effectively be estimating the average or aggregate channel coefficient in each symbol interval. So in the fast-fading case, the channel coefficient  $c_l(i)$  should be understood to represent an average value.

Collecting all received signal samples in the vector  $\mathbf{r}$ , we can write

$$\mathbf{r} = \sum_{l=1}^L \sum_{i=0}^{M-1} c_l(i) \mathbf{x}_l(i) + \mathbf{n} \quad (2)$$

where  $\mathbf{n}$  is a zero-mean Gaussian noise vector with covariance matrix  $N_0 \mathbf{I}$ ,

$$\mathbf{x}_l(i) = \sum_{k=1}^K d_k(i) \begin{bmatrix} 0_{iN+\tau_l} \\ s_k \\ 0_{(M-i-1)N-\tau_l} \end{bmatrix} = \sum_{k=1}^K d_k(i) s_{kl}(i), \quad (3)$$

$s_k^T = [s_k(0), s_k(1), \dots, s_k(N-1)]$ ,  $\mathbf{0}_p$  is the  $p$ -dimensional vector of zeros and  $s_{kl}(i)$  is implicitly defined in (3).

The conventional method of channel estimation proceeds in two steps. First, rough estimates are derived through correlation with the spreading code of the pilot channel, synchronized to the time delay of the desired path. Then, these rough estimates are passed through a smoothing filter (for instance a moving-average filter) to produce smoothed estimates.

The rough estimate of the first channel coefficient  $c_1(i)$  is

$$\hat{c}_1(i) = d_1^*(i) s_{11}^H(i) \mathbf{r} \quad (4)$$

where  $d_1^*(i)$  denotes the complex conjugate of  $d_1(i)$ , a pilot symbol. Substituting (3) into (2), and ignoring inter-symbol interference terms, yields

$$\begin{aligned} s_{11}^H(i)r &= \sum_{l=1}^L c_l(i)d_1(i)s_{11}^H(i)s_{1l}(i) \\ &+ \sum_{l=1}^L \sum_{k=2}^K c_l(i)d_k(i)s_{11}^H(i)s_{kl}(i) + z(i) \end{aligned} \quad (5)$$

where  $z(i) = s_{11}^H(i)n$ . Therefore

$$\begin{aligned} \hat{c}_1(i) &= c_1(i) + \sum_{l=2}^L c_l(i)\rho_{11}(1, l) \\ &+ \sum_{l=1}^L \sum_{k=2}^K c_l(i)d_k^*(i)d_k(i)\rho_{1k}(1, l) + d_1^*(i)z(i), \end{aligned} \quad (6)$$

where  $\rho_{jk}(m, n) = s_{jm}^H(i)s_{kn}(i)$ , i.e. the cross-correlation between the  $m$ th and  $n$ th paths of the  $j$ th and  $k$ th codes. We also assume without loss of generality that  $\|s_{11}(i)\| = 1$ , and note that all spreading-code correlations are time-invariant.

From (6), it is clear that

$$E[\hat{c}_1(i)] = c_1(i) + \sum_{l=2}^L c_l(i)\rho_{11}(1, l) \neq c_1(i) \quad (7)$$

or in other words the rough channel estimates, and hence the smoothed ones derived from them, are biased. When each user transmits over an independent channel, as on the uplink, equation (7) still holds because all ISI and MAI terms have zero mean. Similarly, when ISI is accounted for, the result applies because ISI has zero mean.

### III. I/Q CODE MULTIPLEXED PILOT SYMBOLS

In some cases, pilot bits are transmitted on the quadriphase (Q) carrier, while data is transmitted on the in-phase (I) carrier. Separation of the two channels is accomplished through the use of orthogonal spreading codes for the I and Q channels. Other orthogonal code channels may be used in parallel by the same user, and of course other users can be transmitting simultaneously on other channels.

As MAI terms all have zero mean, they do not contribute to any bias in the channel estimates and it is therefore sufficient for our purposes to consider a simple scenario in which only one user transmits through a multipath channel. The transmitted signal will then be

$$x(n) = \sum_{i=0}^{M-1} b_1(i)s_1(n - iN) + jb_p(i)s_p(n - iN) \quad (8)$$

where  $b_1(i)$  and  $b_p(i)$  denote the data and pilot bits respectively, and  $s_1(n)$  and  $s_p(n)$  are the data and pilot spreading codes respectively, which are orthogonal over one symbol interval.

The received signal vector will be

$$r = \sum_{l=1}^L \sum_{i=0}^{M-1} c_l(i)[b_1(i)s_{1l}(i) + jb_p(i)s_{pl}(i)] + n \quad (9)$$

where

$$s_{1l}(i) = \begin{bmatrix} 0_{iN+\tau_l} \\ s_1 \\ 0_{(M-i-1)N-\tau_l} \end{bmatrix} \text{ and } s_{pl}(i) = \begin{bmatrix} 0_{iN+\tau_l} \\ s_p \\ 0_{(M-i-1)N-\tau_l} \end{bmatrix}$$

$s_1$  and  $s_p$  are defined similarly to  $s_k$  in the previous section.

A correlation-based estimate of  $c_1$  will be

$$\hat{c}_1(i) = -jb_p(i)s_{p1}^T(i)r, \quad (10)$$

because it may be shown that in the absence of noise and multipath interference,  $\hat{c}_1(i)$  calculated in this way will equal  $c_1$ . If we ignore ISI (which will have a mean of zero and not contribute to estimate bias anyway),

$$\begin{aligned} \hat{c}_1(i) &= c_1(i) + \sum_{l=2}^L c_l(i)\rho_{pp}(1, l) \\ &+ \sum_{l=2}^L -jc_l(i)b_p(i)b_1(i)\rho_{p1}(1, l) + z(i). \end{aligned} \quad (11)$$

Here,  $\rho_{pp}(1, l) = s_{p1}^T(i)s_{pl}(i)$  and  $\rho_{p1}(1, l) = s_{p1}^T(i)s_{1l}(i)$ .

The mean of the rough estimate  $\hat{c}_1(i)$  is therefore once again not equal to  $c_1(i)$ , and so is biased.

**Long Codes** When long codes are used, the second term on the right-hand sides of (7) and (11) become zero-mean random variables, and hence the bias disappears. The channel estimate bias is therefore a phenomenon created by the use of short codes, with conventional correlation-based channel estimation. However the bias term increases estimation error, and hence improved performance can be obtained even in long-code systems using the bias removal method discussed next.

### IV. A BIAS-FREE ESTIMATOR

From (7) and (11), we see that in both TDM and I/Q code multiplexed cases, the mean value of the conventional channel estimate is

$$E[\hat{c}(i)] = R_p c(i) \quad (12)$$

where  $\hat{c}(i) = [\hat{c}_1(i), \hat{c}_2(i), \dots, \hat{c}_L(i)]^T$  is the vector of conventional estimates,

$$R_p = \begin{bmatrix} 1 & \rho_{pp}(1, 2) & \dots & \rho_{pp}(1, L) \\ \rho_{pp}(2, 1) & 1 & \dots & \rho_{pp}(2, L) \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{pp}(L, 1) & \dots & \rho_{pp}(L, L-1) & 1 \end{bmatrix}$$

and  $c(i) = [c_1(i), \dots, c_L(i)]^T$ . For the time-multiplexed pilot system of section II,  $\rho_{pp}(m, n)$  should be replaced by  $\rho_{11}(m, n)$ .

To create an unbiased estimate from  $\hat{c}(i)$ , we can "decorrelate" the estimates to form

$$\hat{c}_0(i) = R_p^{-1} \hat{c}(i). \quad (13)$$

The mean value of  $\hat{c}_0(i)$  will be equal to  $c$ , and hence it is an unbiased channel estimate. For short codes,  $R_p$  is time-invariant and its inverse only needs to be computed once. Thus the additional complexity in performing the decorrelation operation effectively comes only from the  $L^2$  multiplications needed in (13).

For long codes, we will either have to re-compute  $R_p$  at each symbol interval, or implement the decorrelation operation using linear multi-stage successive interference cancellation [9].

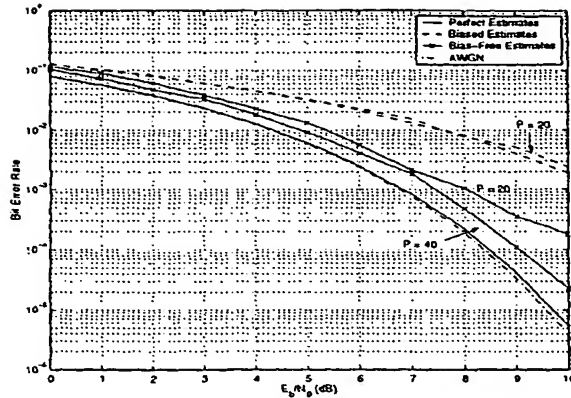


Figure 1: Bit error rates for Rake receivers with conventional and bias-free channel estimates, in a short-code system. The curve labelled "AWGN" is for an AWGN channel, without ISI.  $P$  is the number of pilot symbols in a frame.

## V. SIMULATIONS AND CONCLUSIONS

The exact bit error rate of a linear receiver in a time-invariant multipath AWGN channel is readily computed as the sum of a number of  $Q$  functions (see for instance [10, Chap. 3]). Using this technique, we obtained the BER curves of Rake receivers with conventional (biased) and bias-free channel estimators for the I/Q code-multiplexed pilot scheme by

1. Averaging the channel estimates over  $P$  pilot symbols;
2. Finding the BER using those channel estimates;
3. Repeating the process over 50 independent frames, and computing the average BER obtained.

For a system with a processing gain of 8, one user and a four-path channel having path delays 0, 2, 4 and 6 chips, the BER for  $P = 20$  and  $P = 40$  are shown in Figure 1. The pilot spreading code was Walsh code 0, and the data spreading code was Walsh code 1. A scrambling code of  $[-1, +1, +1, -1, -1, +1, +1, -1]$  was applied to both data and pilot channels. In this case, we see a gap of over 2 dB between the performance using biased and bias-free estimates at a BER of  $10^{-3}$ . Also, while the performance of the bias-free algorithm improves with a larger number of pilot symbols per frame, the same cannot be said for the conventional algorithm, since the errors in the latter case are due mostly to estimation bias and not to noisy estimates.

The difference in channel estimation bias is even more vividly brought out in Figure 2, where we compare the conventional and decorrelating estimators. 100 independent trials were conducted, each of which selected a different set of spreading codes and channel coefficients. A three-path channel was used, and the processing gain was eight. The  $E_b/N_0$  was fixed at 10 dB throughout, and the error magnitudes were obtained by averaging over 500 pilot symbols in each trial. We note that the error in the conventional estimator case is some ten times larger than the decorrelating estimator's.

A final note is that the performance differences between the two algorithms are not always as large as in Figure 1.

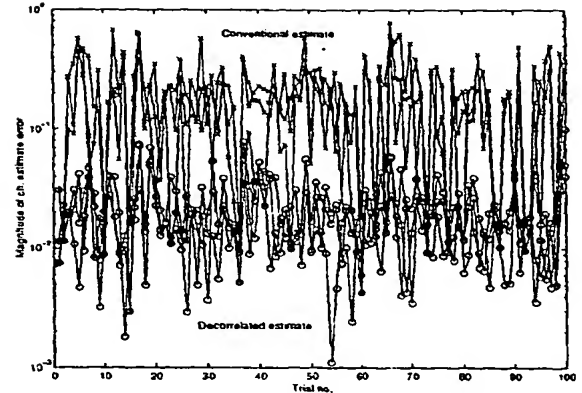


Figure 2: Magnitude of the channel estimate errors for 100 independent channels, using the conventional and decorrelating estimators.

Depending on the channel coefficients and the pilot spreading codes used, the bias terms in (7) and (11) may be so small that no discernible differences in performance can be observed. However, the bias-free estimates always perform better than the conventional estimates.

We have shown in this paper that when short spreading sequences are used in CDMA, channel estimation done in the conventional correlation-based way will lead to biased estimates. In simulations, we have observed that this bias can lead to a substantial performance penalty in a Rake receiver. The performance loss in a multiuser detector has not been quantified and may be investigated in the future. To remove this channel estimate bias, we proposed a simple scheme akin to decorrelating multiuser detection. From simulations, we saw that the new bias-free channel estimation algorithm reduced the magnitude of estimation errors by a factor of about ten.

## REFERENCES

- [1] J. G. Proakis, *Digital Communications*, McGraw-Hill, 3rd edition, 1995.
- [2] M. Honig, U. Madhow, and S. Verdú, "Blind adaptive multiuser detection," *IEEE Trans. Inf. Theory*, vol. 41, no. 4, pp. 944-960, July 1995.
- [3] D. Guo, L. K. Rasmussen, and T. J. Lim, "Linear parallel interference cancellation in long-code CDMA multiuser detection," *IEEE J. Sel. Areas in Comms.*, vol. 17, no. 12, pp. 2074-2081, Dec. 1999.
- [4] S. D. Gray, M. Kocic, and D. Brady, "Multiuser detection in mismatched multiple-access channels," *IEEE Trans. Communications*, vol. 43, no. 12, pp. 3080-3089, Dec. 1995.
- [5] F. C. Zheng and S. K. Barton, "On the performance of near-far resistant CDMA detectors in the presence of synchronization errors," *IEEE Trans. Communications*, vol. 43, no. 12, pp. 3037-3045, Dec. 1995.
- [6] M. Torlak and G. Xu, "Blind multiuser channel estimation in asynchronous CDMA systems," *IEEE Trans. Signal Processing*, vol. 45, no. 1, pp. 137-147, Jan. 1997.
- [7] T. J. Lim and S. Roy, "Adaptive filters in multiuser CDMA detection," *Baltzer Wireless Networks*, vol. 4, no. 4, pp. 307-318, 1998.

- [8] Y. Song and S. Roy, "Blind adaptive reduced-rank detection for DS-CDMA signals in multipath channels," *IEEE J. Sel. Areas in Communications*, vol. 17, no. 11, pp. 1960–1970, Nov. 1999.
- [9] L. K. Rasmussen, T. J. Lim, and A.-L. Johansson, "A matrix-algebraic approach to successive interference cancellation in CD-MA," *IEEE Trans. Communications*, vol. 48, no. 1, pp. 145–151, Jan. 2000.
- [10] S. Verdú, *Multisuser Detection*, Cambridge Univ. Press, Cambridge, UK, 1998.